

FJ started a 31 day treatment program which involved daily injections of a medication. The first day's injection was 18 mg, and each subsequent day's injection was 6% less than the previous day's injection. Find the total amount of the injections. SCORE: ____ / 5 PTS

Your final answer may NOT use ... NOR $\sum$. It may use +, -, $\times$, $\div$ and powers. (It does NOT need to be simplified into a single number.)

a_n = AMOUNT OF INJECTION ON n^{th} DAY

$$a_1 = 18$$

$$a_2 = 18(1 - 0.06) = 18(0.94)$$

$$a_3 = 18(0.94)^2$$

$\vdots$

$$a_n = 18(0.94)^{n-1}$$

$$\text{TOTAL} = \frac{18(1 - 0.94^{31})}{1 - 0.94}$$

(Handwritten annotations: ① above 18, ②½ above 31, ①½ below 1 - 0.94)

Use Pascal's Triangle to expand and simplify $(2n^4 - r^3)^5$.

SCORE: ____ / 5 PTS

$$(2n^4)^5 + 5(2n^4)^4(-r^3) + 10(2n^4)^3(-r^3)^2 + 10(2n^4)^2(-r^3)^3 + 5(2n^4)(-r^3)^4 + (-r^3)^5$$

$$= \underbrace{32n^{20}}_{\textcircled{\frac{1}{2}}} - \underbrace{80n^{16}r^3}_{\textcircled{1}} + \underbrace{80n^{12}r^6}_{\textcircled{1}} - \underbrace{40n^8r^9}_{\textcircled{1}} + \underbrace{10n^4r^{12}}_{\textcircled{1}} - \underbrace{r^{15}}_{\textcircled{\frac{1}{2}}}$$

Using the formulae for the sums of the powers of integers in your textbook, find the sum $\sum_{n=1}^{310} (2n^2 - 5n + 3)$. SCORE: _____ / 4 PTS

Your final answer may **NOT** use ... **NOR** $\sum$. It may use $+$, $-$, $\times$, $\div$. (It does **NOT** need to be simplified into a single number.)

$$2 \sum_{n=1}^{310} n^2 - 5 \sum_{n=1}^{310} n + \sum_{n=1}^{310} 3$$

$$= \underbrace{2 \cdot \frac{310(311)(621)}{6}}_{\textcircled{2}} - \underbrace{5 \cdot \frac{310(311)}{2}}_{\textcircled{1\frac{1}{2}}} + \underbrace{310 \cdot 3}_{\textcircled{\frac{1}{2}}}$$

Use mathematical induction to prove that 3 is a factor of $n^3 + 6n^2 + 5n$ for all positive integers n .

SCORE: ____ / 10 PTS

BASIS STEP: 3 IS A FACTOR OF $1^3 + 6(1)^2 + 5(1) = 12$ ①

INDUCTIVE
STEP

① ASSUME 3 IS A FACTOR OF $k^3 + 6k^2 + 5k$
FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$

MUST HAVE WORD
"INTEGER"
① ①

[PROVE 3 IS A FACTOR OF $(k+1)^3 + 6(k+1)^2 + 5(k+1)$]

$$\underline{(k+1)^3 + 6(k+1)^2 + 5(k+1)} \quad ①$$

$$= k^3 + 3k^2 + 3k + 1 \\ + 6k^2 + 12k + 6 \\ + 5k + 5$$

$$= (k^3 + 6k^2 + 5k) + (3k^2 + 15k + 12)$$

$$= \underline{(k^3 + 6k^2 + 5k) + 3(k^2 + 5k + 4)} \quad ②$$

SINCE 3 IS A FACTOR OF BOTH $(k^3 + 6k^2 + 5k)$
AND $3(k^2 + 5k + 4)$ ①

THEREFORE 3 IS A FACTOR OF $(k+1)^3 + 6(k+1)^2 + 5(k+1)$ ①

SO, 3 IS A FACTOR OF $n^3 + 6n^2 + 5n$ ① ②

FOR ALL INTEGERS $n \geq 1$ ②

Find the rational number representation of the repeating decimal $0.2\overline{63}$ using the method discussed in lecture.

SCORE: ____ / 6 PTS

NOTE: Only the 63 is repeated.

$$\underline{0.2 + 0.063 + 0.00063 + 0.0000063 + \dots}$$
$$= \frac{2}{10} + \frac{0.063}{1 - \frac{1}{100}} \quad (1)$$

$$= \frac{2}{10} + \left[\frac{\frac{63}{1000}}{\frac{99}{100}} \right] \quad (2\frac{1}{2})$$

$$= \frac{2}{10} + \frac{63}{1000} \cdot \frac{100}{99}$$

$$= \left(\frac{1}{2} \right) \left[\frac{2}{10} + \frac{7}{110} \right] \quad (1)$$

$$= \left[\frac{29}{110} \right] \quad (1)$$